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Growth Facts and Solow Model (II/II)

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- ▶ Recap of what we discussed in the last lecture.
- ▶ The basics of the Solow model and its implications are the learning goals.
- ▶ "Why are some countries richer than others?"
    - ▶ Revisiting the question with more data and new concepts.
- ▶ Conclusion.

Recap: Economic thinking

► The economy is too complex to understand altogether.
► We want to understand and intervene.
  ► Economic policy aims:
► We need to communicate how to choose the best option.
  ► … systematic thinking.
  ► … to observe the economy and lay out the facts.
  ► … to interpret the data and establish possible mechanisms.
  ► … to compare the costs and choose what we have to give up.
  ► … a clear mapping between the conditions, interventions, and outcomes.

- ▶ Why are some countries richer than others?
- ▶ Are some countries richer because they had a head start?
- ▶ Can we develop a country through donations?
- ▶ Is growth easier for countries with smaller populations?

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Recap: Solow model in words

▶ Output is produced using capital, labor, enhanced by productivity.

▶ A constant fraction of output is saved and invested in capital.

▶ Capital depreciates over time.

▶ Labor force size changes.

▶ Productivity improves over time.

▶ These forces determine how output per worker evolves.

◁◁

$$\text{Production: } Y_t = F(A_t, K_t, L_t)$$

$$\text{Savings: } sY_t = I_t$$

$$\text{Investment and Depreciation: } K_{t+1} = K_t + I_t - \delta K_t$$

$$\text{Labor force change: } L_{t+1} = L_t + nL_t$$

$$\text{Productivity improvement: } A_{t+1} = A_t + aA_t$$

$$\text{Output per worker progress: } y_{t+1} = G(y_t, s, \delta, n, a)$$

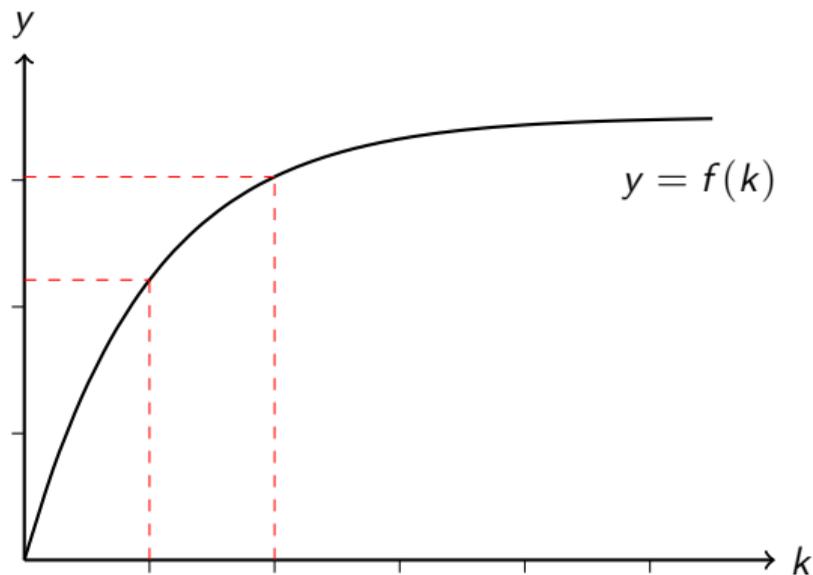Production: $y_t = f(k_t) = A k_t^{\alpha}$

Equilibrium condition: $s y_t = i_t$

Capital accumulation: $k_{t+1}(1 + n) = (1 - \delta)k_t + i_t$

Output per worker evolution: $y_{t+1} = G(y_t, s, \delta, n)$

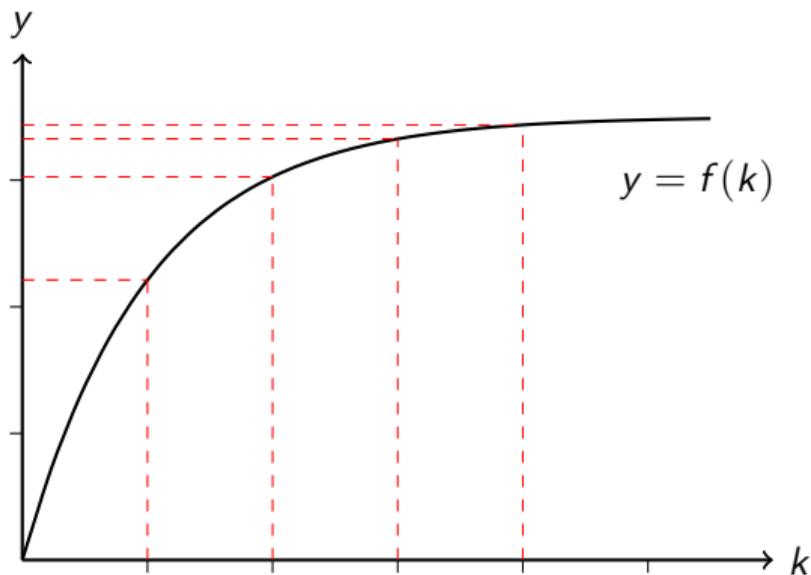Steady state: $y^* = \left(\dfrac{s}{\delta + n} \right)^{\frac{\alpha}{1-\alpha}}$

Which mechanism in the model provides that there is a steady state?

y

$y = f(k)$

k

▶ When k is low,
additional capital significantly raises output.

Diminishing marginal product

- ▶ When k is low,
  additional capital significantly raises output.

- ▶ When k is high,
  extra capital raises output slightly.

- ▶ Imagine two countries.
  One is richer, and the other is poorer.

Another big question

Do poorer economies "catch up" to richer ones over time?

- ▶ If true, income gaps shrink.
- ▶ If not, gaps can persist (or even widen).

Intuition: Why might poorer economies grow faster?

Think of capital per worker (k): machines, tools, and infrastructure per person.

- ▶ When k is low, **an extra machine helps a lot**.
- ▶ When k is high, **an extra machine helps less**.

Key idea

Because of diminishing returns, countries with lower k tend to have higher returns on investment, so they can grow faster.

Convergence: Two Versions

1) Absolute (Unconditional) Convergence

All countries converge to the <u>same</u> long-run income level.

2) Conditional Convergence

Countries converge to their <u>own</u> long-run income levels, depending on:

- ▶ saving/investment rates,
- ▶ education/human capital,
- ▶ population growth,
- ▶ institutions and policies.

▶ In the real world, evidence fits **conditional convergence** better.

What the Solow Model Says

In the Solow model, output per worker depends on capital per worker:
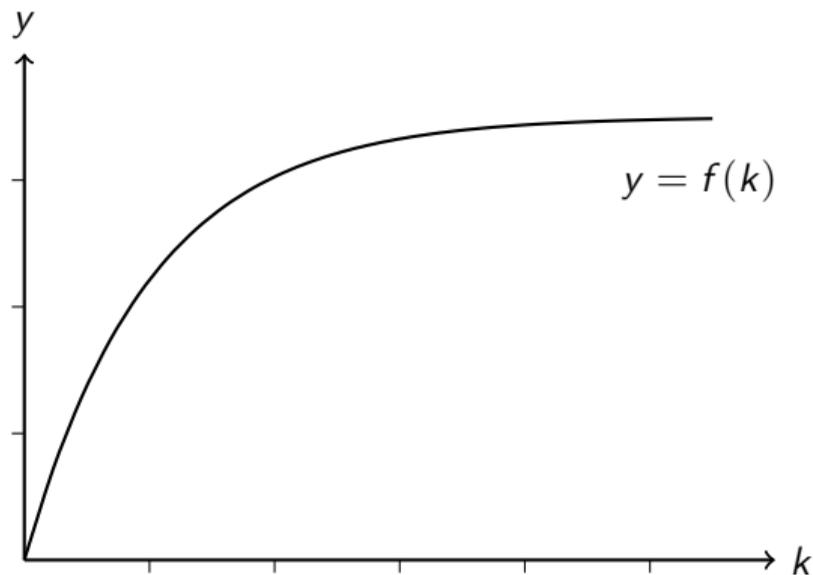
$$y = f(k)$$

Investment increases k, depreciation and population growth reduce k "per person".

Steady state idea

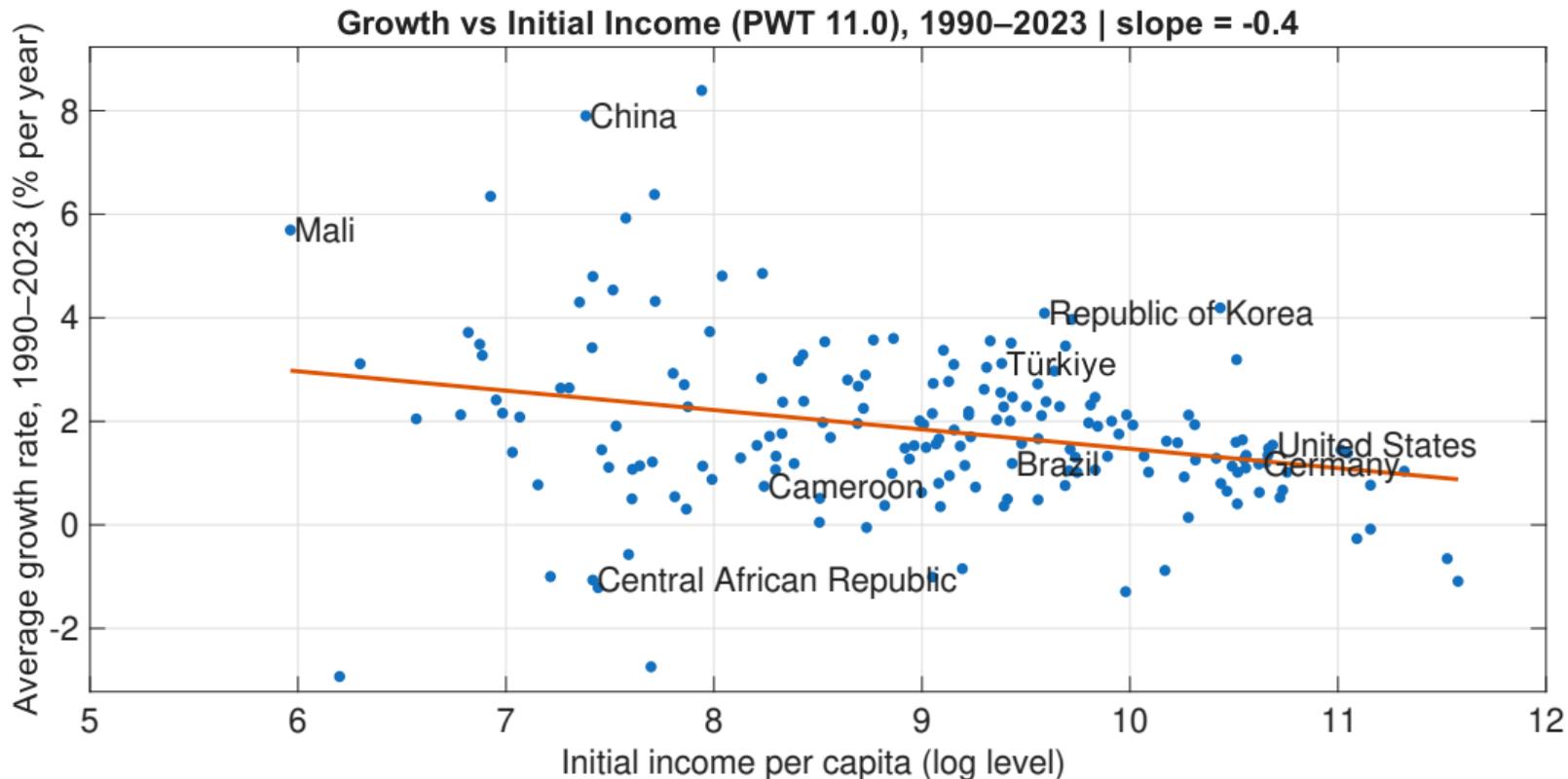Each economy moves toward a long-run level (k^*, y^*).

- ▶ If you start **below** y^*, you grow faster.
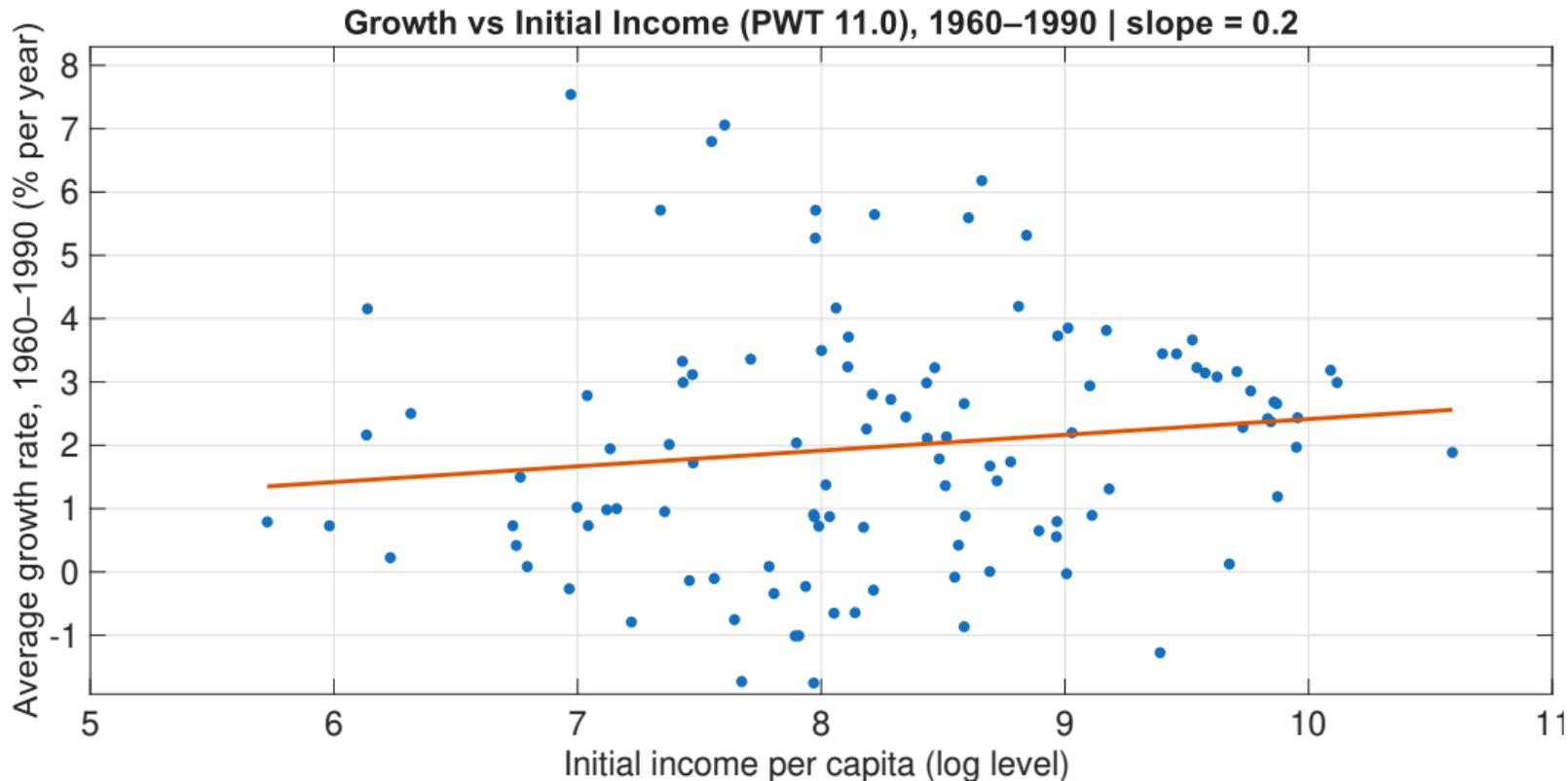- ▶ Growth slows down as you get closer to y^*.

Diminishing Returns

- ▶ When k is low,
  additional capital significantly raises output.

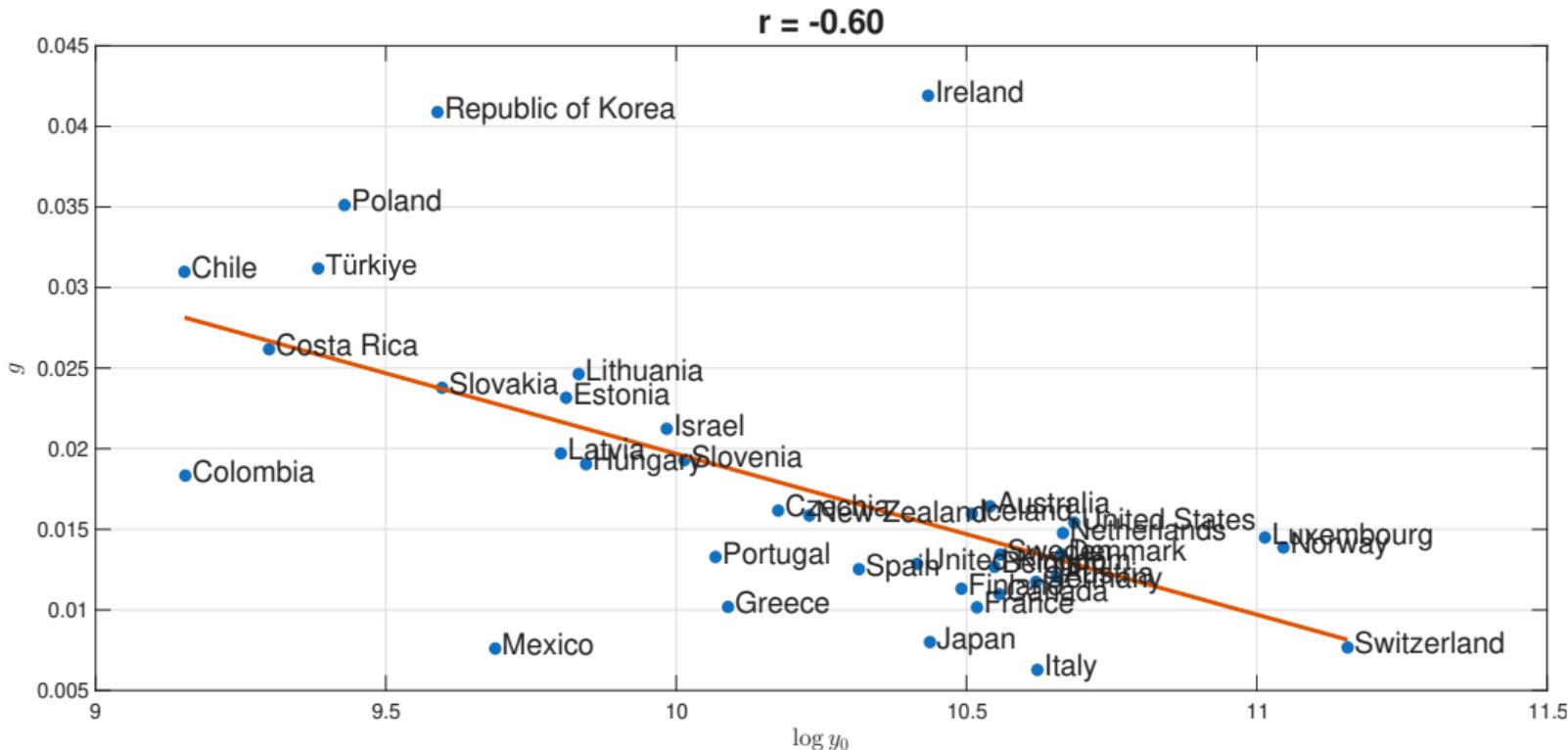- ▶ When k is high,
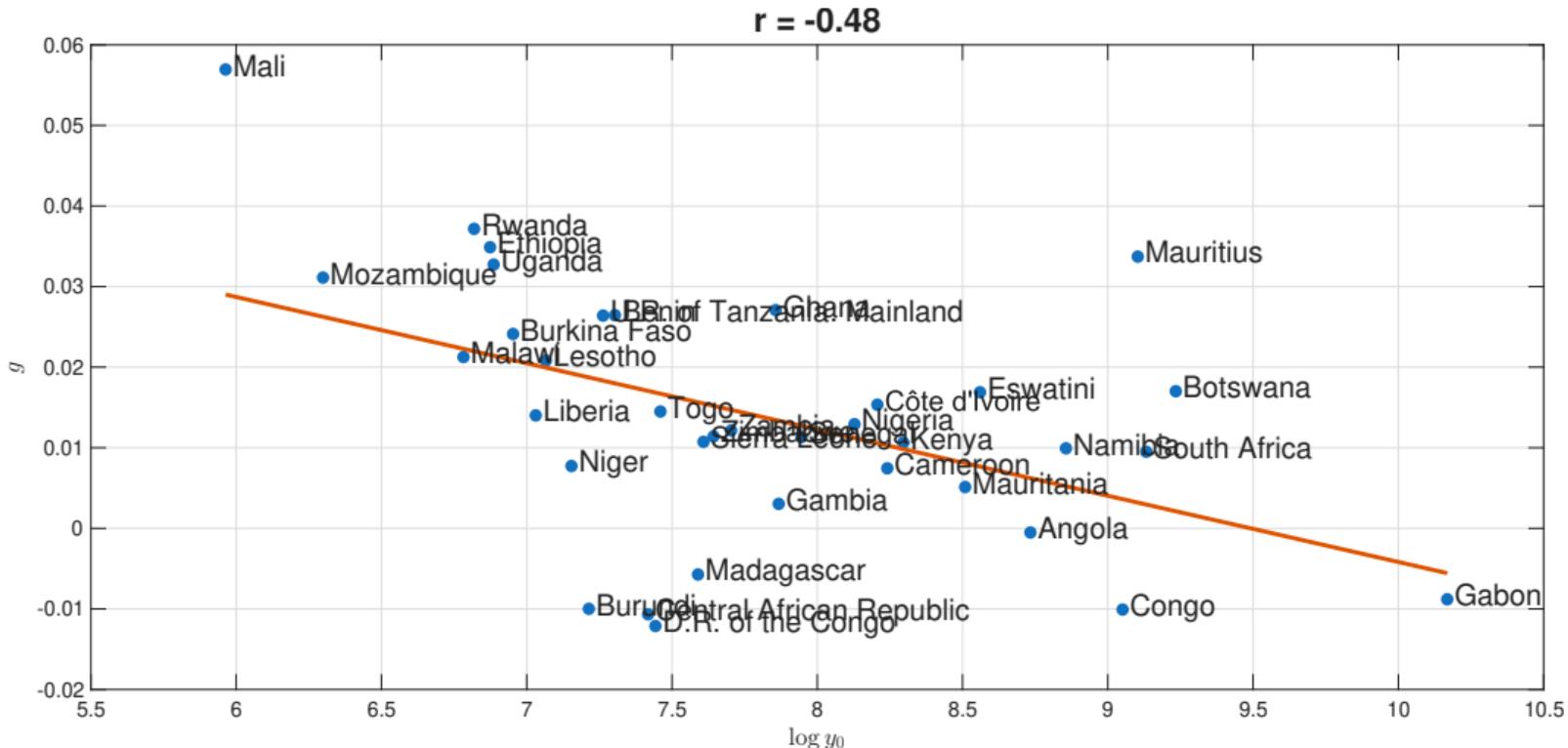 extra capital raises output slightly.

Growth vs Initial Income (PWT 11.0), 1990–2023 | slope = -0.4
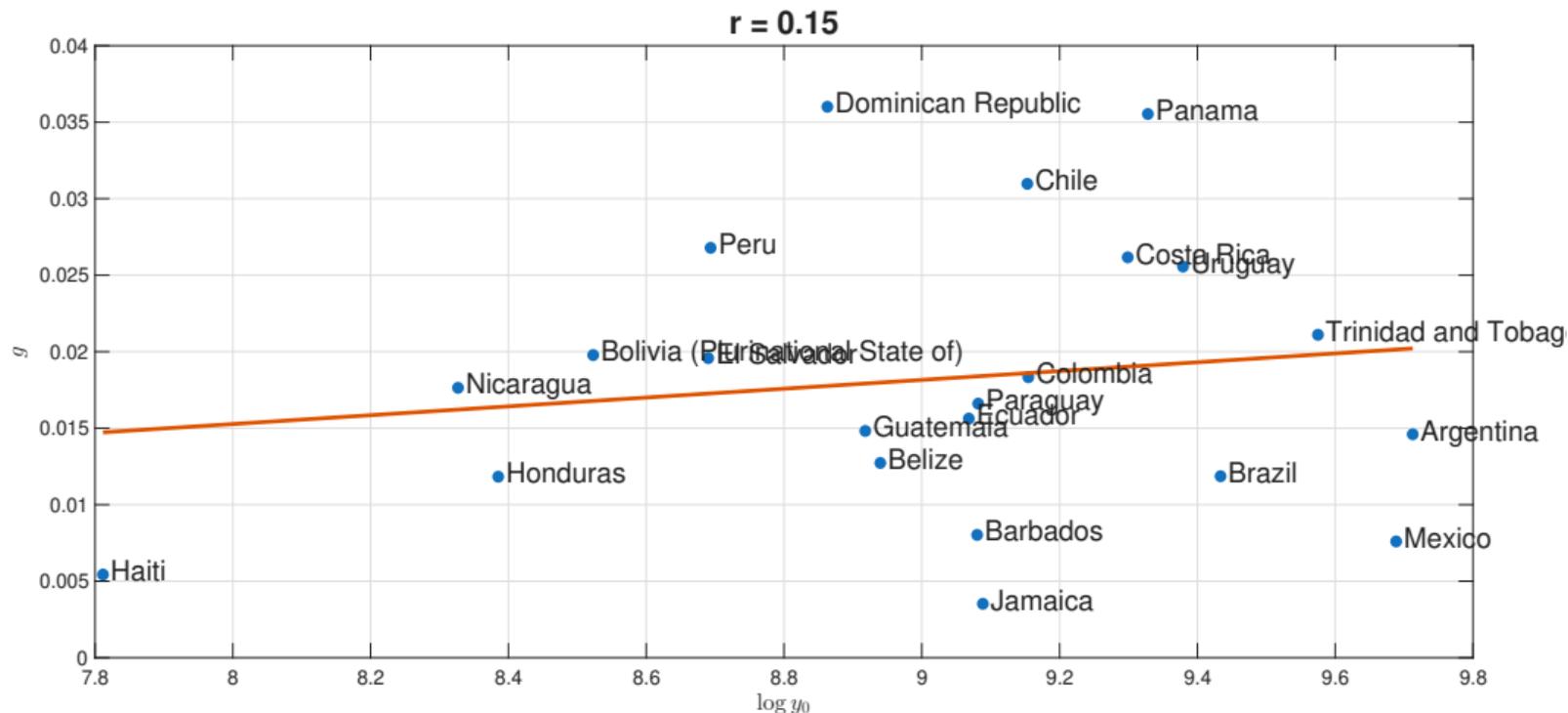
Growth vs Initial Income (PWT 11.0), 1960–1990 | slope = 0.2

Club convergence: OECD (1990-2023)

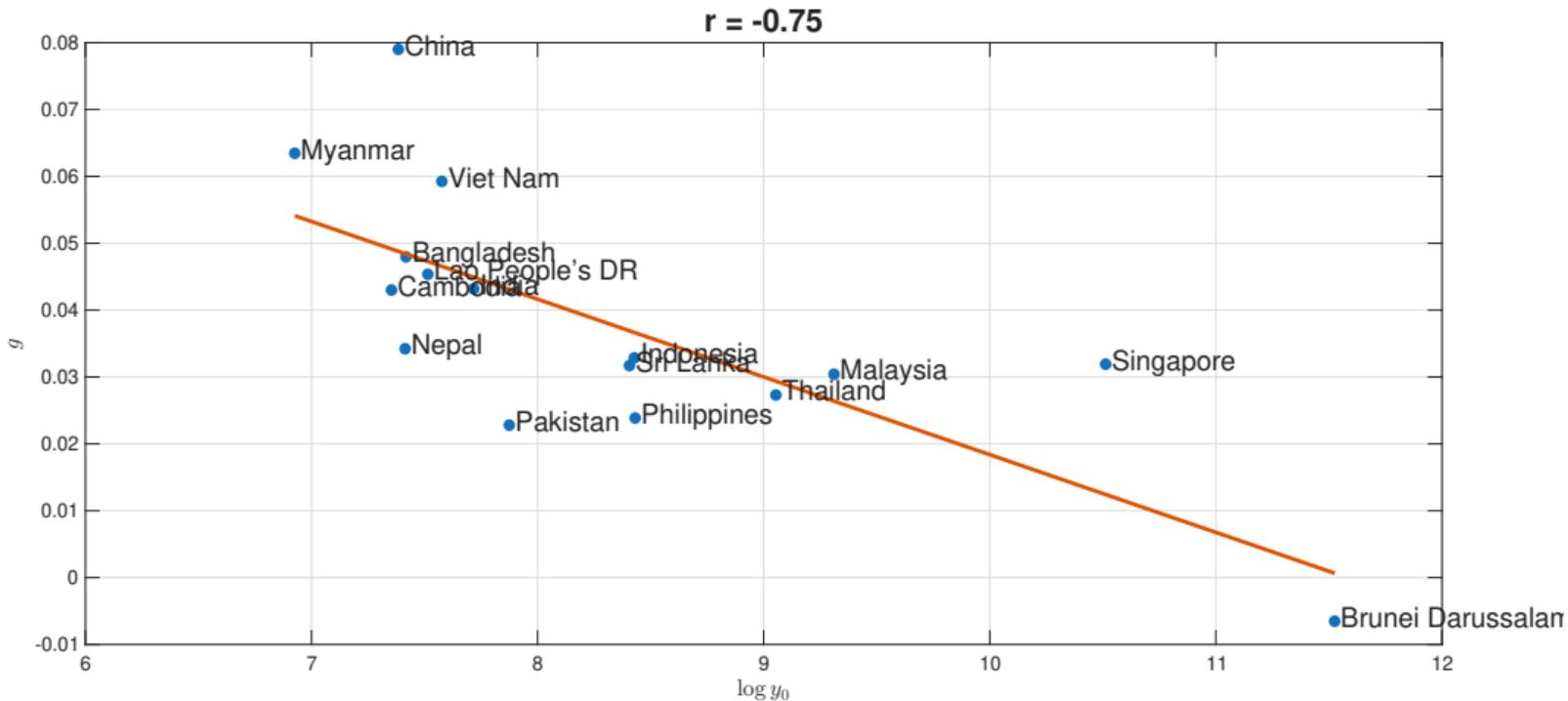# Club convergence: Sub-saharan countries (1990-2023)

Club convergence: Latin America (1990-2023)
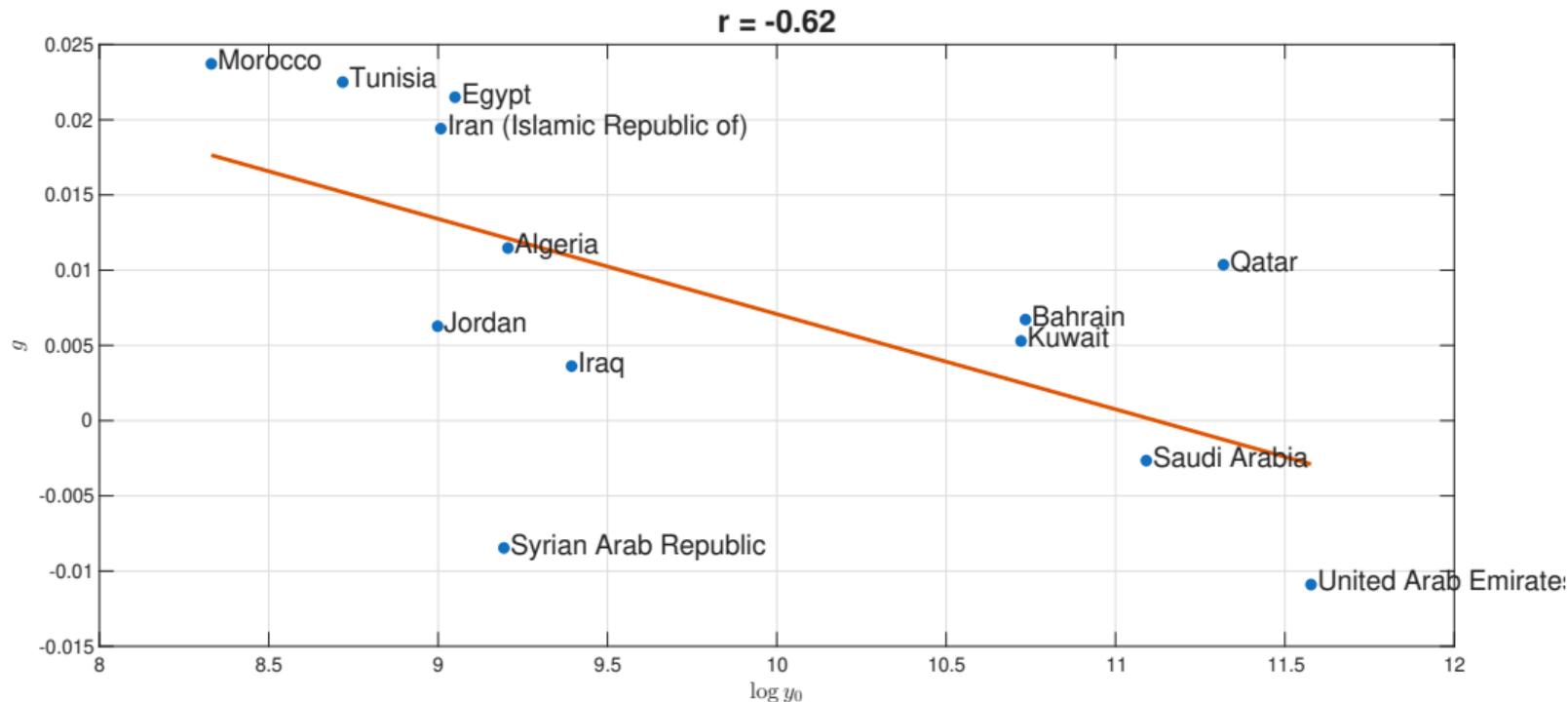

r = 0.15
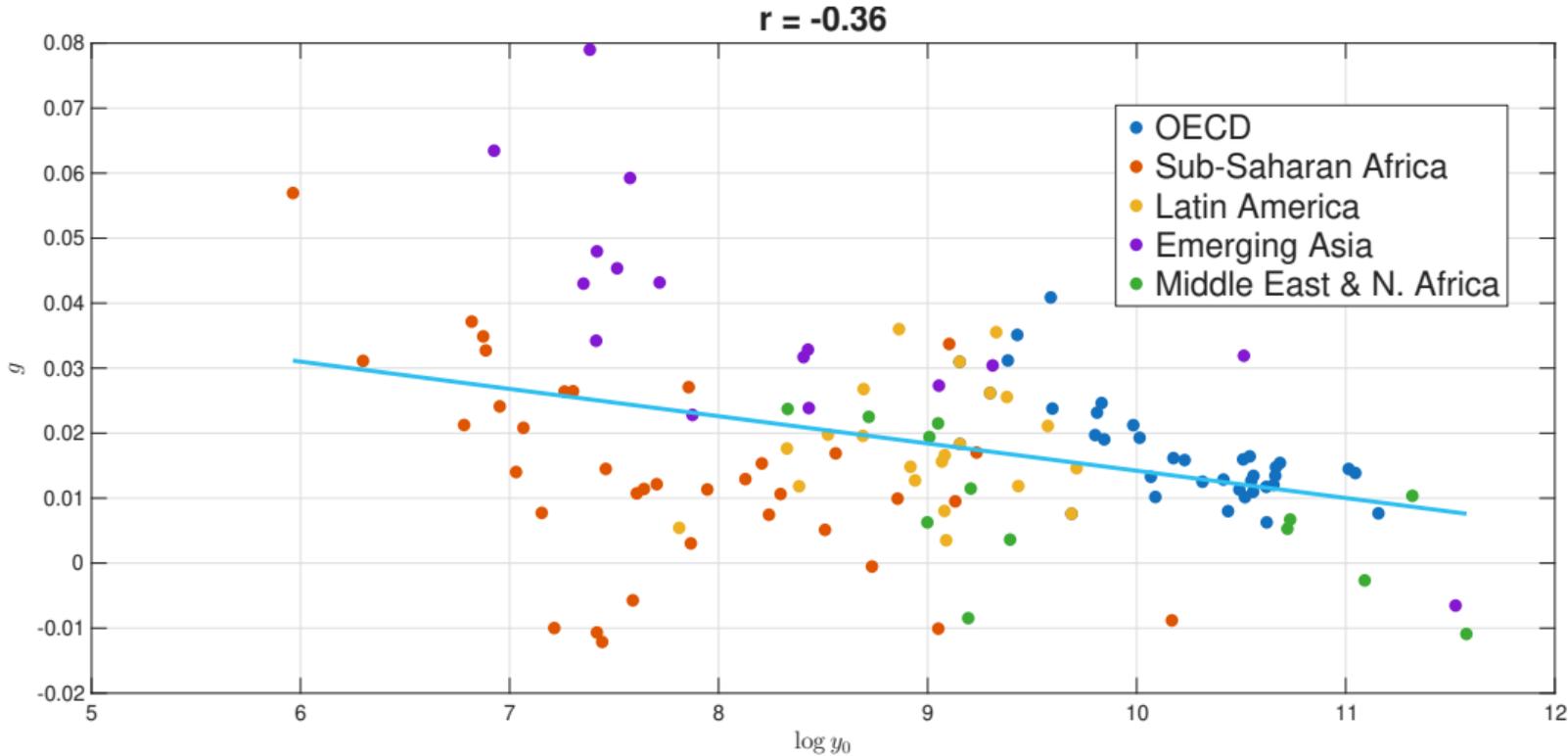
Venezuela excluded.

Club convergence: Emerging Asia (1990-2023)

Club convergence: Middle East & North Africa (1990-2023)

Yemen excluded.

Absolute convergence (1990-2023)

Augmented Solow Model with Human Capital

$$Y = K^{\alpha} H^{\phi} (AL)^{1-\alpha-\phi}$$

Steady-state output per worker:

$$y^* = A \left(\frac{s_k}{n+a+\delta} \right)^{\frac{\alpha}{1-\alpha-\phi}} \left(\frac{s_h}{n+a+\delta} \right)^{\frac{\phi}{1-\alpha-\phi}}$$
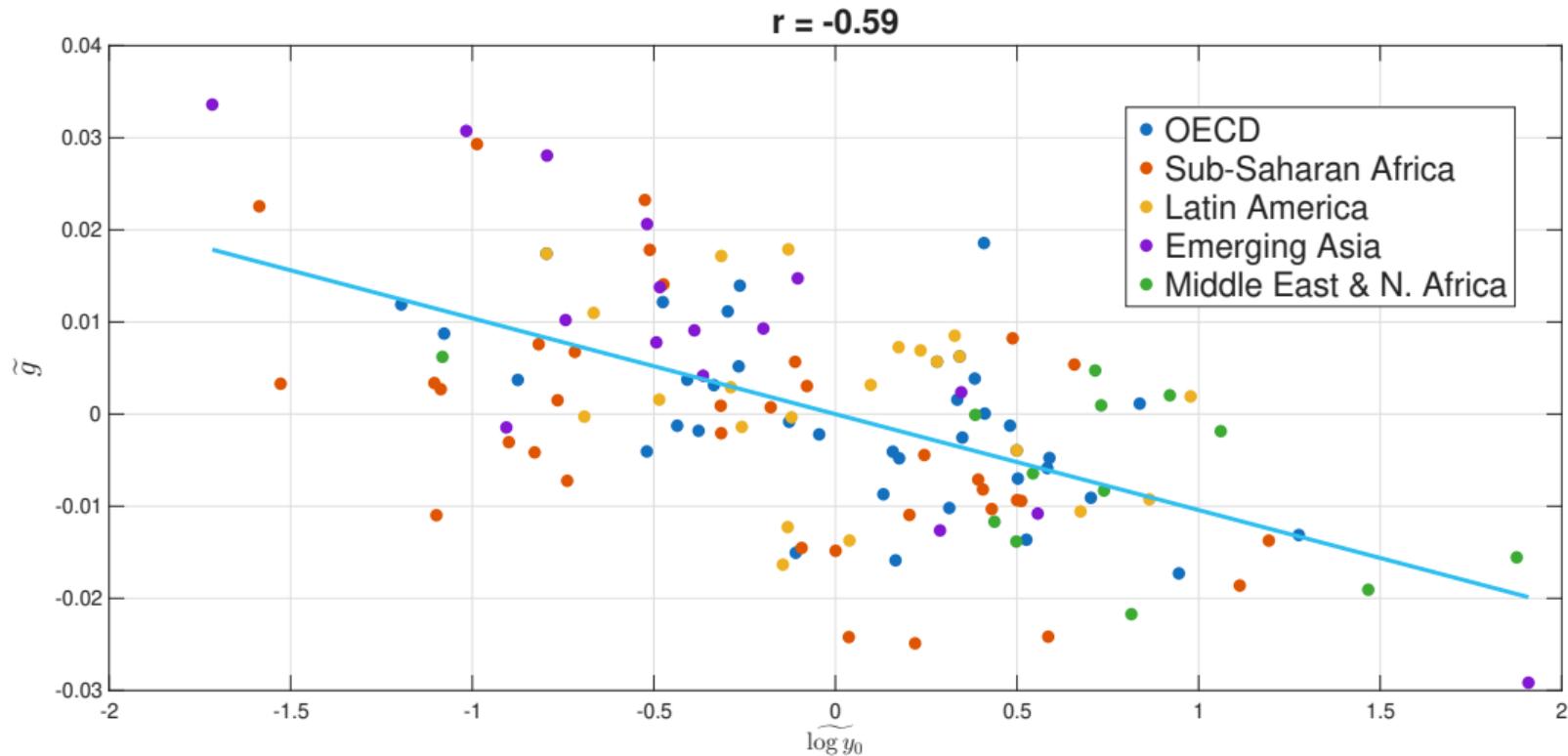
Key idea: The steady-state level of income depends on:

- Physical capital saving rate (s_k)
- Human capital investment (s_h)
- Population growth (n)

Conditional convergence

Countries with <u>similar</u> savings, education, and population growth converge at the <u>same steady-state income level</u>.

Conditional convergence

Controlled factors (theory vs. data):

▶ Physical capital saving rate (s_k) → share of GDP devoted to investment

▶ Human capital (H or s_h) → human capital index based on years of schooling

▶ Population growth (n) → population growth rate

▶ Depreciation rate (δ) → delta (average depreciation rate of capital)

Note: These are empirical approximations of theoretical concepts.

▶ Why not use data alone? Why do we still need economic models?

▶ Economic models also help when we cannot quantify objects.

    ▶ or when the data are not enough to understand chicken and egg situations.

- ► Technology and its absorption differences
- ► Trade openness
- ► Institutional differences
- ► Political instability
- ► Commodity cycles
- ► Measurement error

What does the evidence say?

- Across the whole world: weak evidence for **absolute convergence**.
- Among similar economies (e.g., within OECD countries), there is stronger evidence for **conditional convergence**.

Takeaway

People catch up faster when they share similar fundamentals (education, investment, institutions).

Why might convergence fail?

- Low savings/investment $\Rightarrow$ slow capital accumulation.
- Low education/human capital $\Rightarrow$ low productivity.
- Poor institutions (weak property rights, instability) $\Rightarrow$ low incentives.
- Conflict, crises, bad policies can create setbacks.

One-slide summary

- Solow model predicts **catch-up** due to diminishing returns (Solow, 1956).
- Real-world evidence supports mostly **conditional convergence**.
- Convergence is more visible among economies with similar fundamentals (Barro & Sala-i-Martin, 1992; Mankiw, Romer & Weil, 1992).

► Aggregate production function: The function that maps inputs (factors of production) into outputs (GDP).

► Returns to scale (RTS): the proportional change in output resulting from an x-fold proportional increase in all inputs (factors of production), evaluated relative to that same x-fold increase.

  ► Constant RTS: $F(2X) = 2Y$
  ► Increasing RTS: $F(2X) > 2Y$
  ► Decreasing RTS: $F(2X) < 2Y$

► Capital accumulation: The process by which an economy increases its capital stock (machines, equipment, buildings, infrastructure) over time through investment.

Key concepts (II/II)

- Equilibrium: A situation in which no agent has an incentive to change their behavior, given the behavior of others.
- Steady state: An unchanging equilibrium over time.
    - Equilibrium condition in the Solow model: $sy_t = i_t$
    - Steady state: $sy^* = i^* = (n + \delta)k^*$
- Diminishing marginal product: As more of a factor of production is added, holding other factors constant, the additional output produced by each extra unit becomes smaller.
- Convergence: Poorer economies catch up to richer ones over time.
    - Absolute convergence: All countries converge to the same steady state level of output per capita.
    - Conditional convergence: Countries converge to their own steady state level of output per capita.

References

Barro, R. J., and Sala-i-Martin, X. (1992). Convergence. *Journal of Political Economy*, 100(2), 223–251.

Mankiw, N. G., Romer, D., and Weil, D. (1992). A Contribution to the Empirics of Economic Growth. *Quarterly Journal of Economics*, 107(2), 407–437.

Solow, R. M. (1956). A Contribution to the Theory of Economic Growth. *Quarterly Journal of Economics*, 70(1), 65–94.