



Bilkent University

Department of Economics

Growth Facts and Solow Model (I/II)

Mahmut S. İpek

Bilkent University

Ankara

February 10, 2026

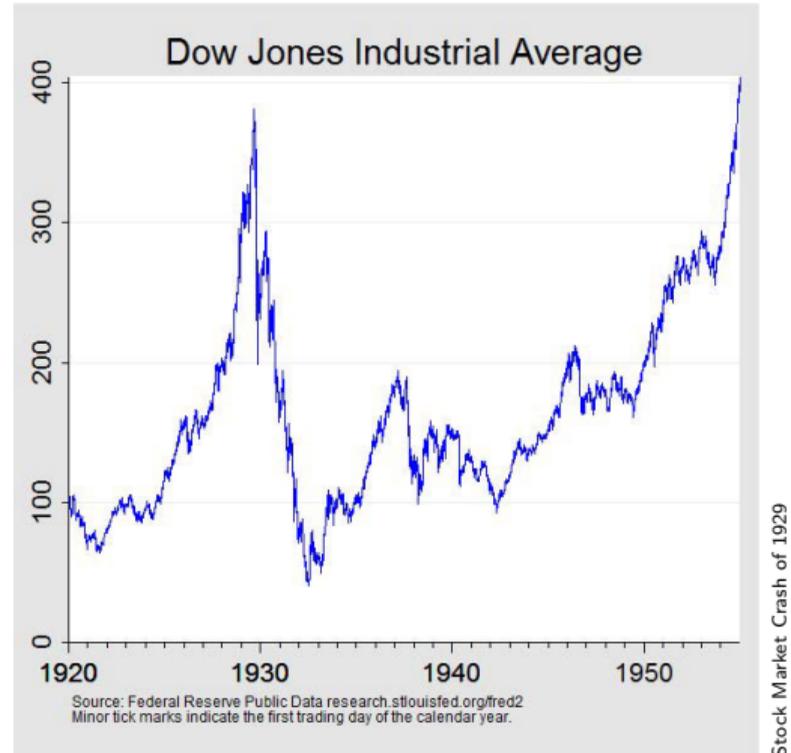
- ▶ How do economists think? At least, macroeconomists.

For the interested: "The Macroeconomist as Scientist and Engineer" by N. Mankiw

- ▶ History for the fun of it.
 - ▶ The basics of systematic economic thinking are the learning goals.
- ▶ "Why are some countries richer than others?"
 - ▶ Exploration of the question and the first substantial answer.

Economic thinking

- ▶ We want to change the world.
Almost romantically!



Economic thinking

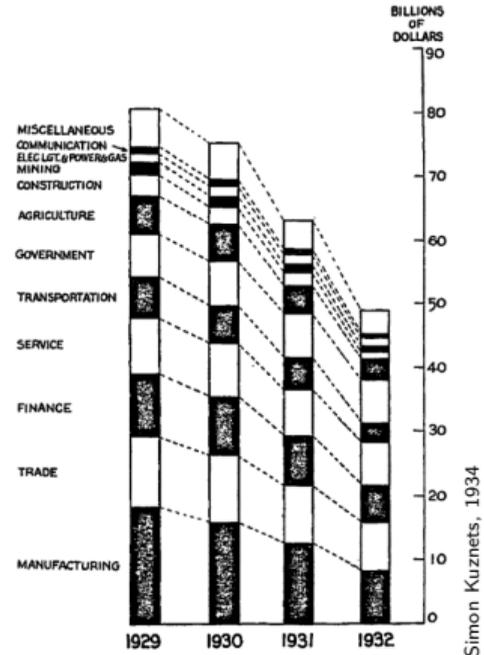
- ▶ We want to change the world.



Early 1930s, Chicago

Behind these numbers are real lives.

INCOME PAID OUT BY INDUSTRIES



Simon Kuznets, 1934

Economic thinking

- ▶ We want to change the world.
- ▶ But the world is complex.



Two models of the planet earth serve two different purposes.
Both are correct; both are wrong.

- ▶ We want to change the world.
- ▶ But the world is complex.
 - ▶ So is the economy.
- ▶ How do macroeconomics help?
 - ▶ Economic policy aims:
“high employment, stable prices, and rapid growth”

Social Progress Index

BASIC NEEDS

- Nutrition & Medical Care
Do people have enough food to eat & receive basic medical care?
- Water & Sanitation
Can people drink water & keep themselves clean without getting sick?
- Housing
Do people have adequate housing with basic utilities?
- Safety
Do people feel safe?

FOUNDATIONS OF WELLBEING

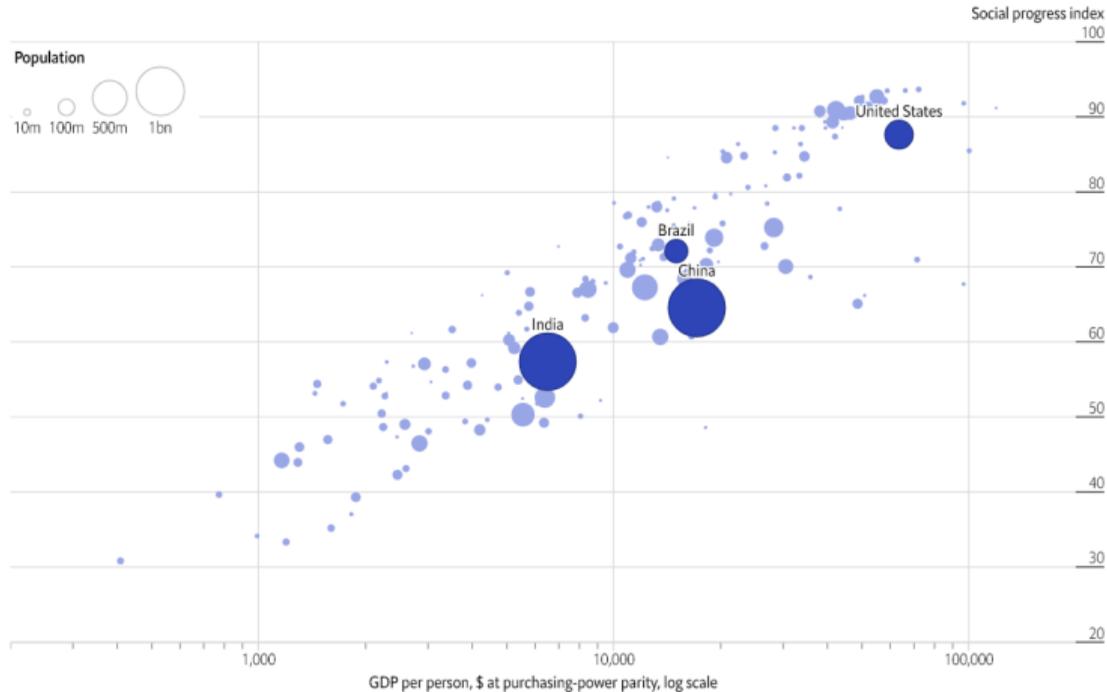
- Basic Education
Do people have access to an educational foundation?
- Information & Communications
Can people freely access ideas & information from anywhere in the world?
- Health
Do people live long & healthy lives?
- Environmental Quality
Does the environment support societal well-being?

OPPORTUNITY

- Rights & Voice
Are people's rights as individuals protected?
- Freedom & Choice
Are people free to make their own life choices?
- Inclusive Society
Is no one excluded from the opportunity to be a contributing member of society?
- Advanced Education
Do people have access to the world's most advanced knowledge?

- “The Empirics of Social Progress” by Fehder, Porter, and Stern (2018)
- Social Progress Index Webpage

GDP vs Social Progress Index



Sources: "Social progress index time series: measuring social outcomes 1990-2020", by J. Harnacek and P. Krylova, Social Progress Imperative, 2023; IMF World Economic Outlook

How well does your country provide for its citizens? (The Economist)

Economic Thinking

- ▶ We want to change the world.
- ▶ But the world is complex.
 - ▶ So is the economy.
- ▶ How do macroeconomics help?
 - ▶ Economic policy aims:
“high employment, stable prices, and rapid growth”
- ▶ We need to communicate how to choose the best option.
 - ▶ Systematic thinking.
 - ▶ What are the given conditions?
 - ▶ How do we interpret those conditions?
 - ▶ Then, let's choose *this!*
 - ▶ But at what cost? Trade-offs.
 - ▶ Clear assumptions and mechanisms.



“Why are some countries richer than others?”



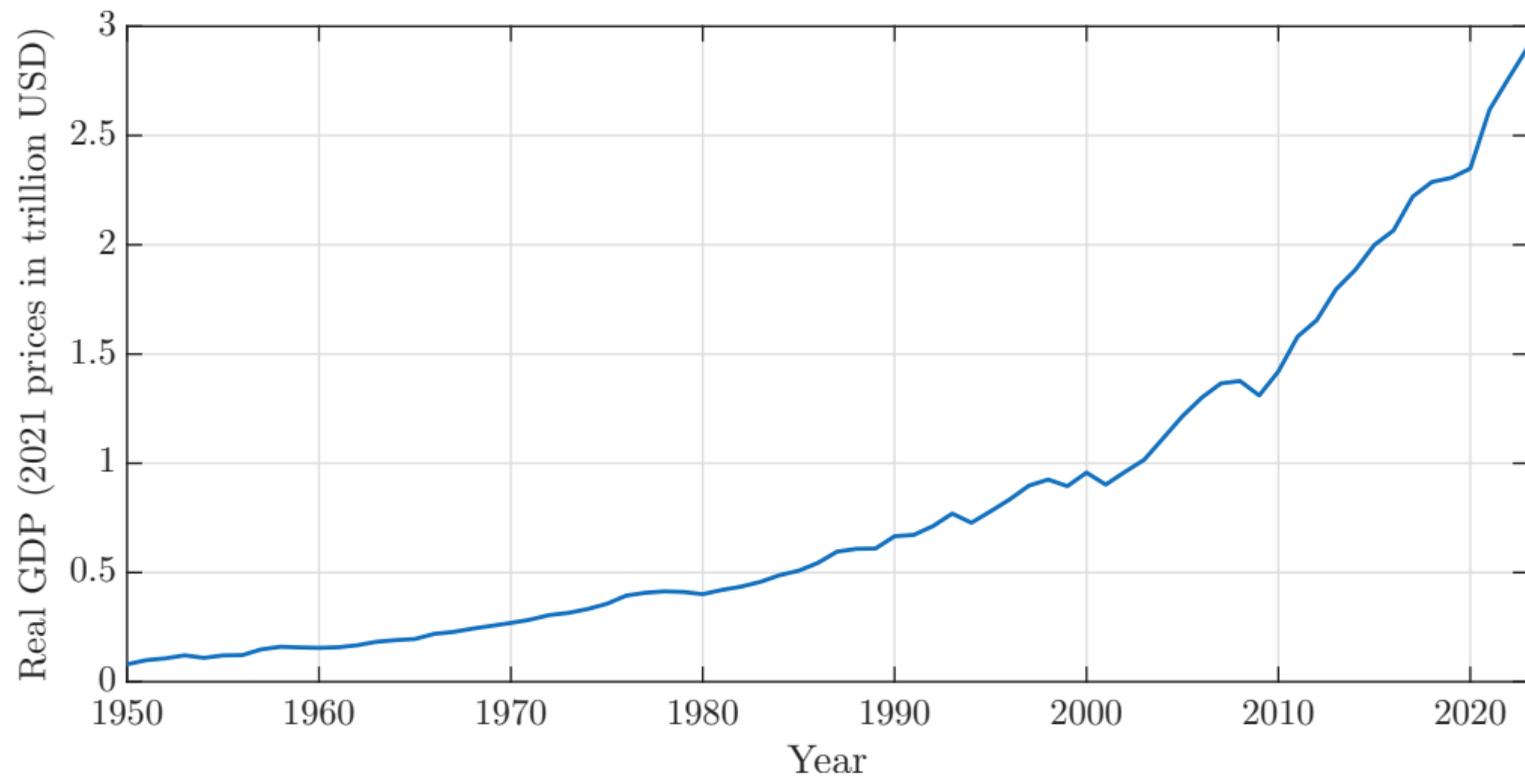
Robert E. Lucas Jr. (1937-2023)

“Once you start thinking about [economic] growth, it is hard to think about anything else.”

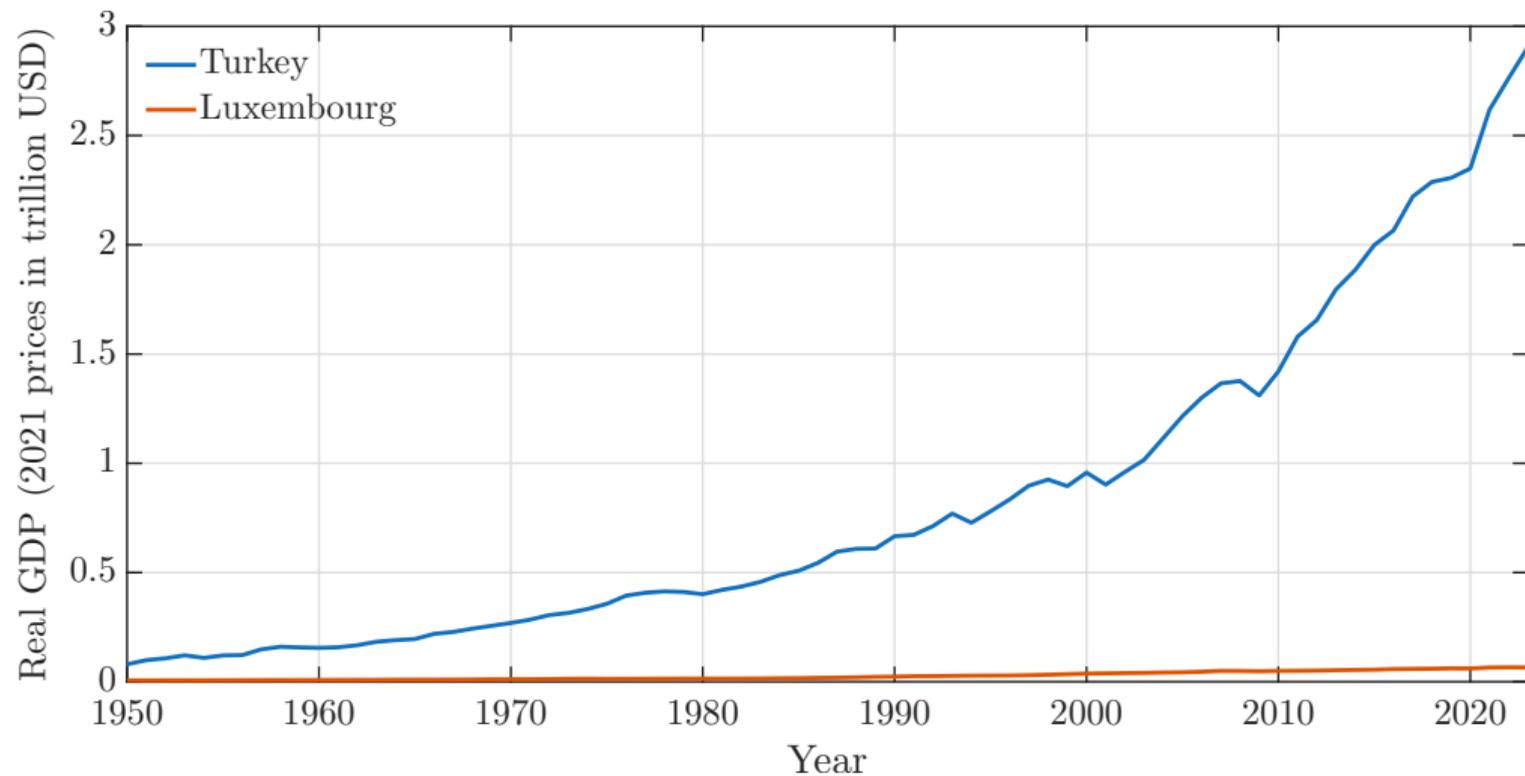
For the interested:

- “Learning from Lucas” by Thomas J. Sargent
- “Nobel Laureate Robert E. Lucas, Jr.: Architect of Modern Macroeconomics” by V. V. Chari

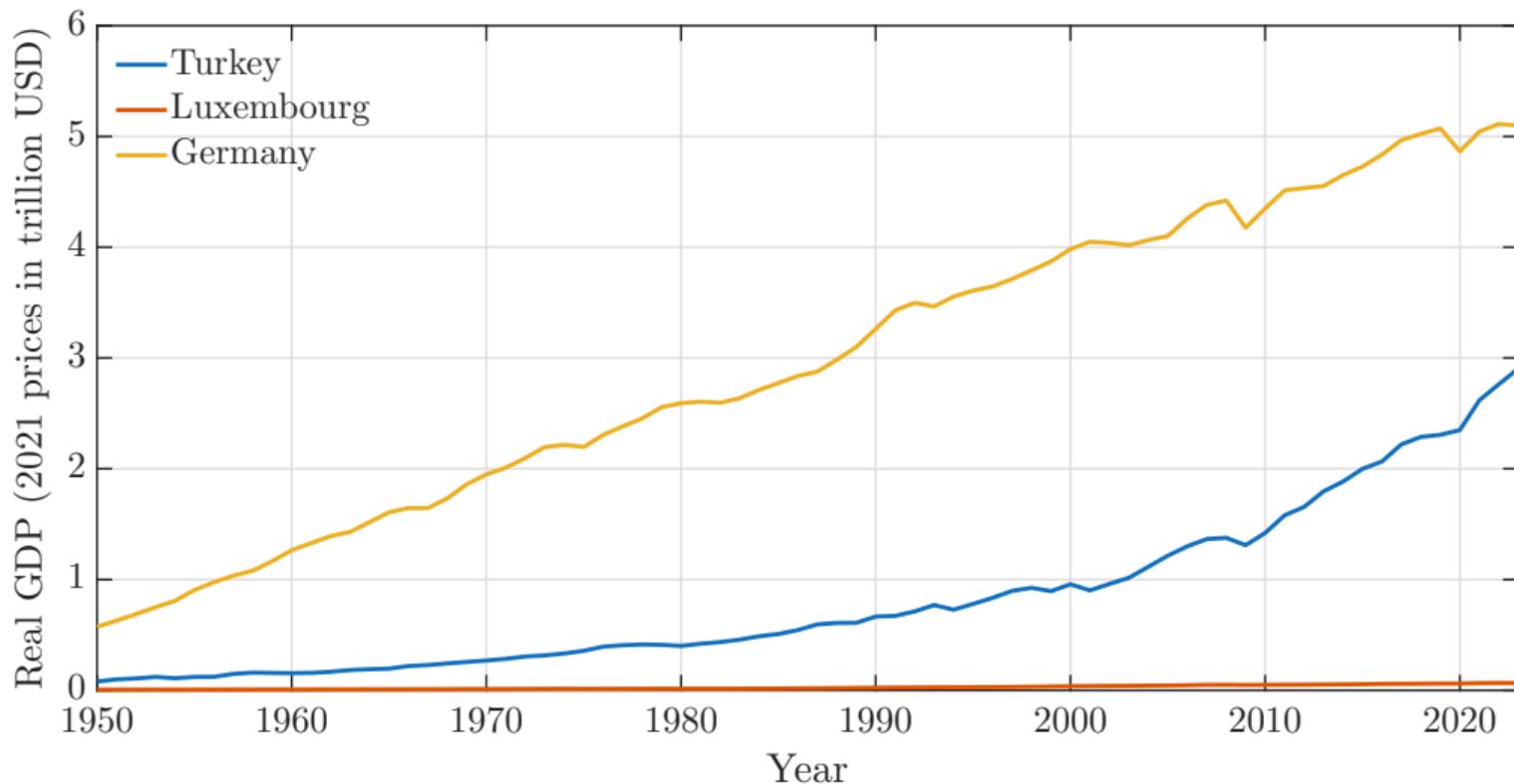
Real GDP: Turkey



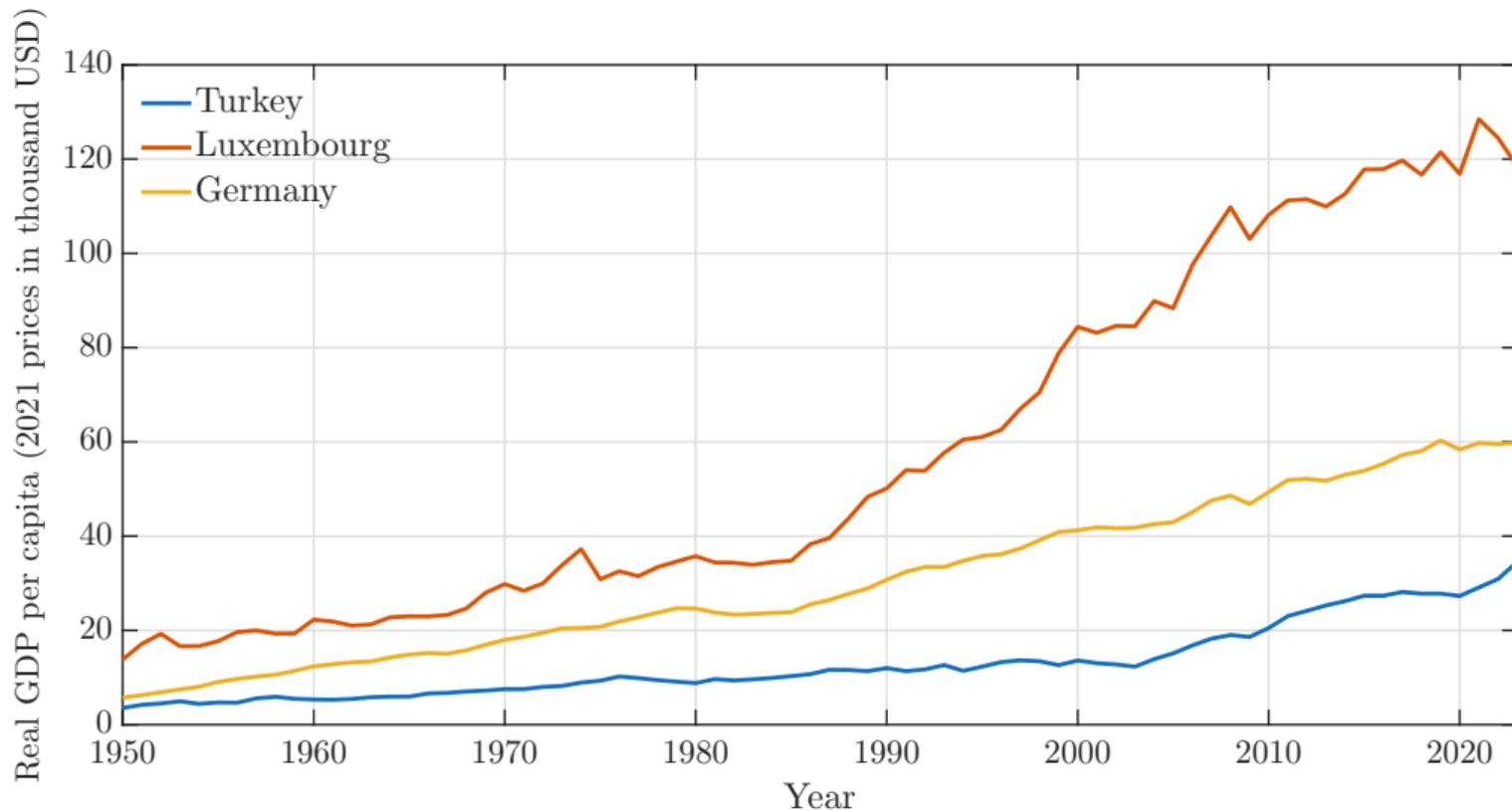
Real GDP: Turkey vs. Luxembourg



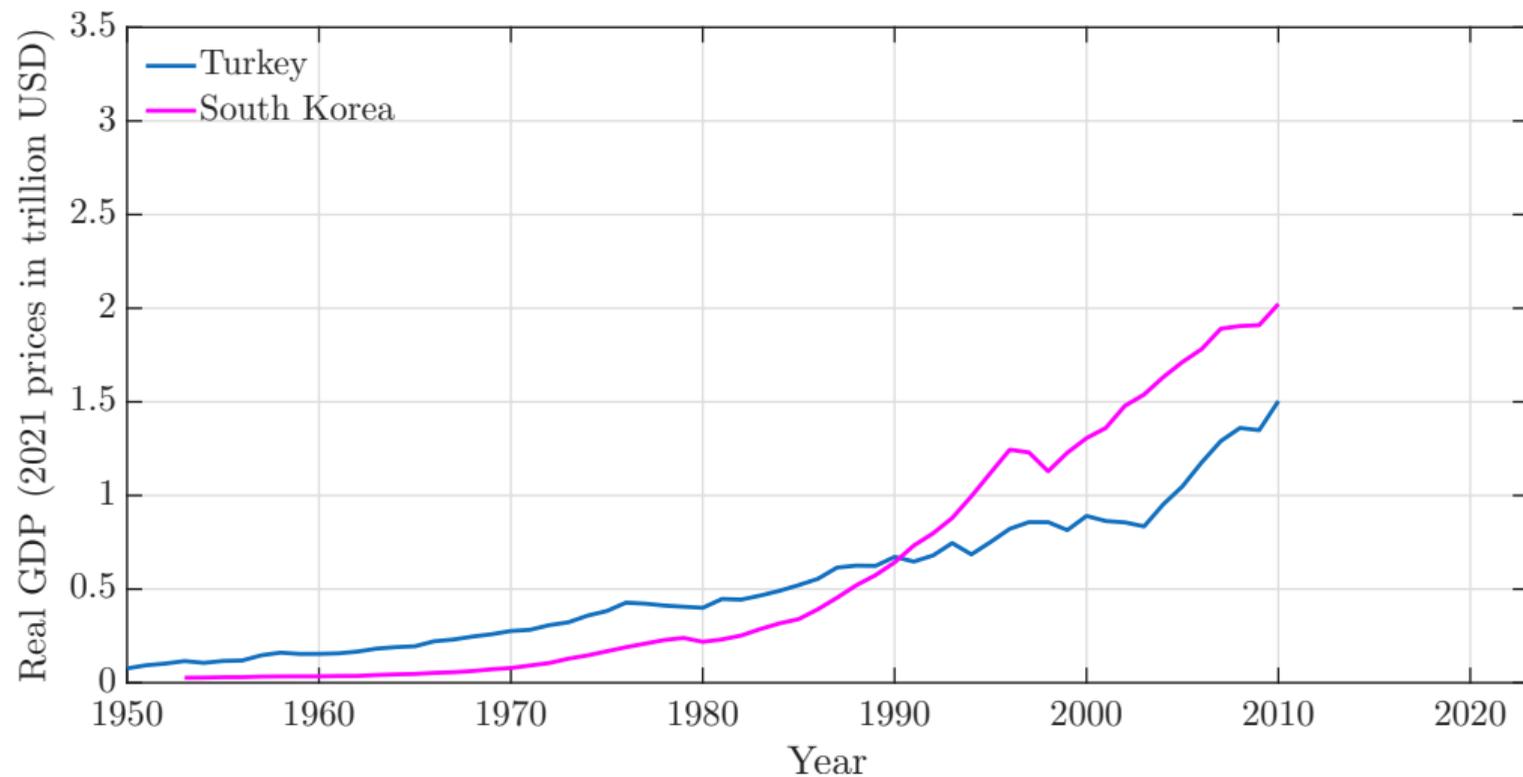
Real GDP: Selected countries



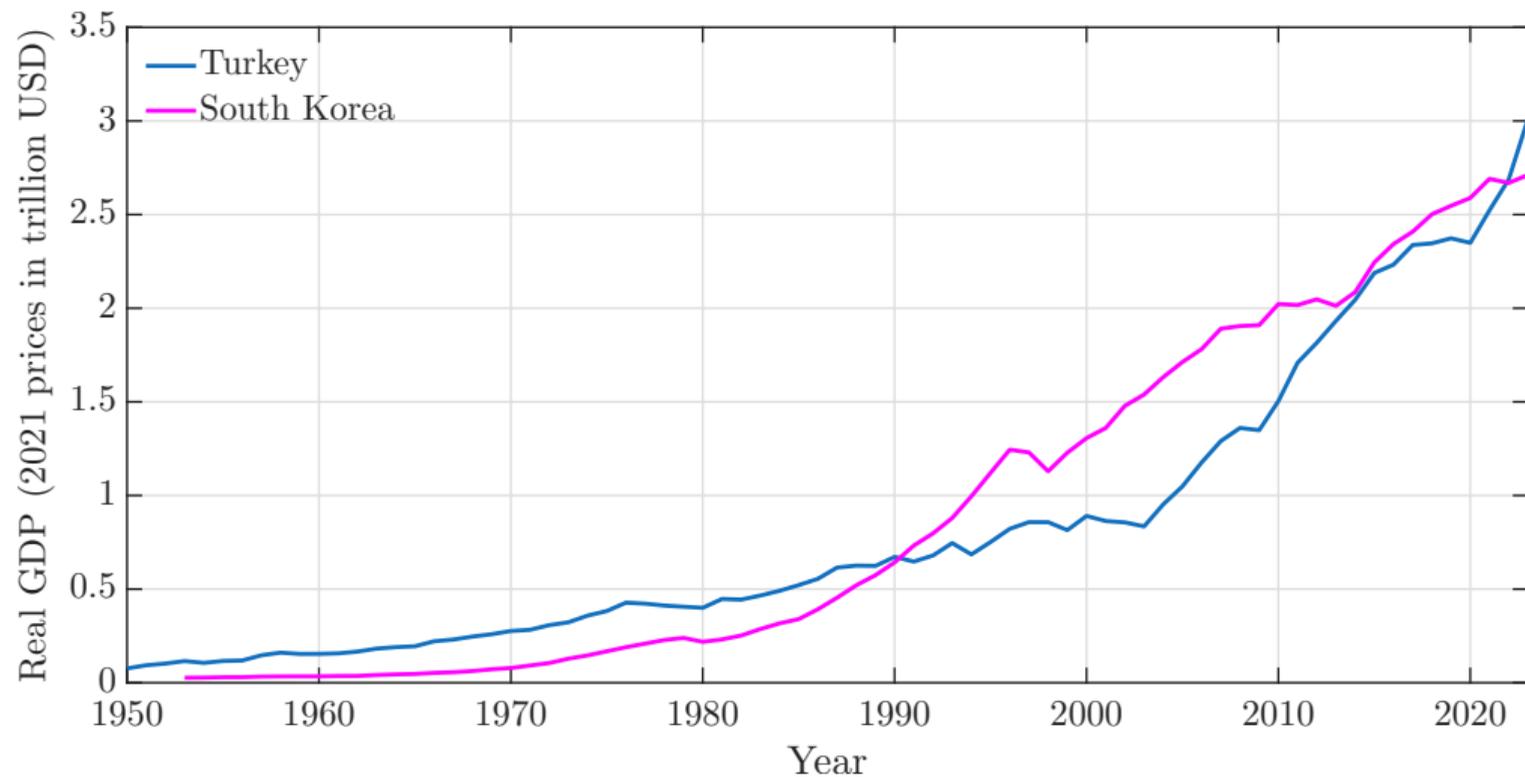
Real GDP per capita: Selected countries



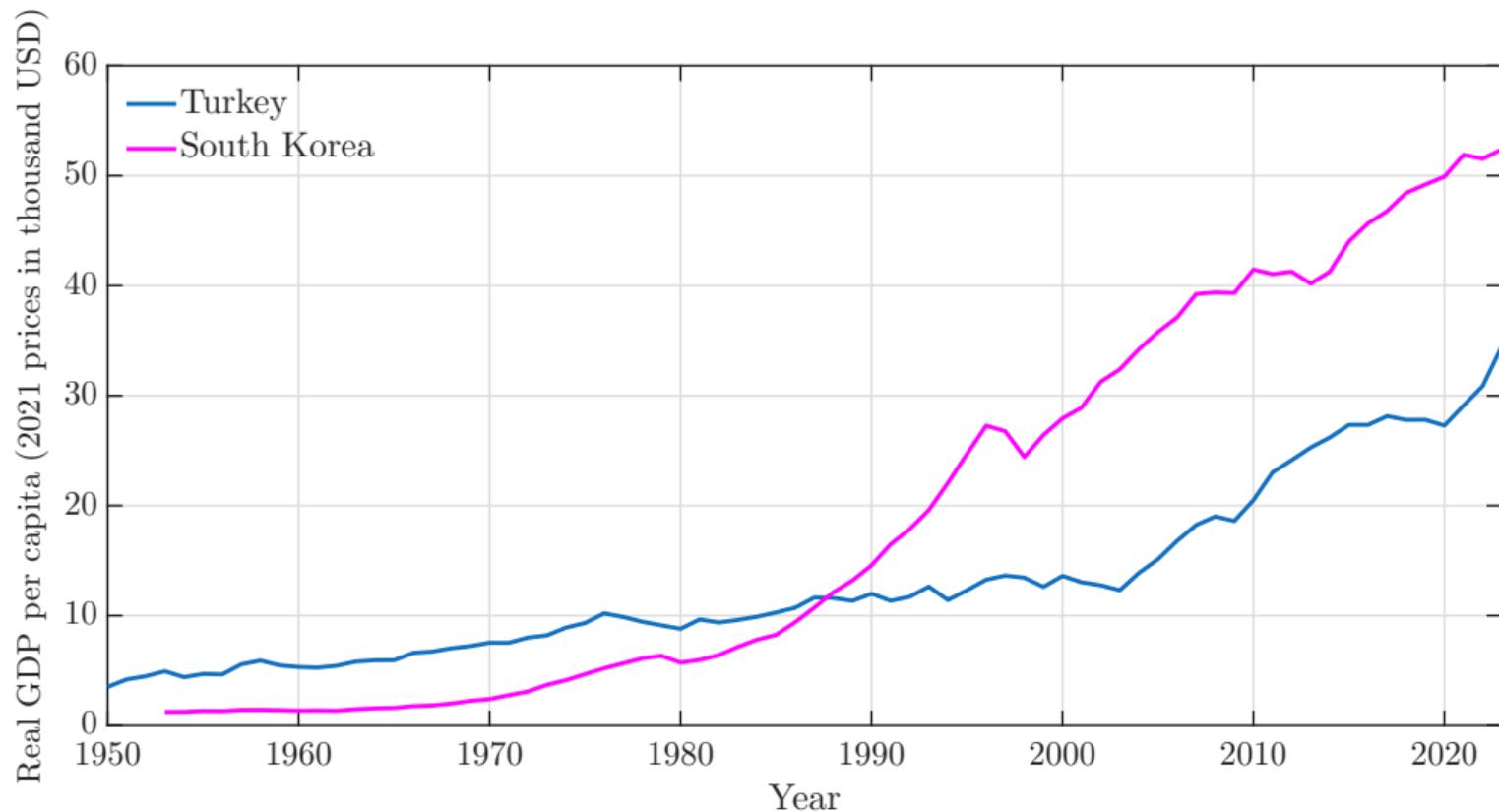
Real GDP: Turkey vs. Korea



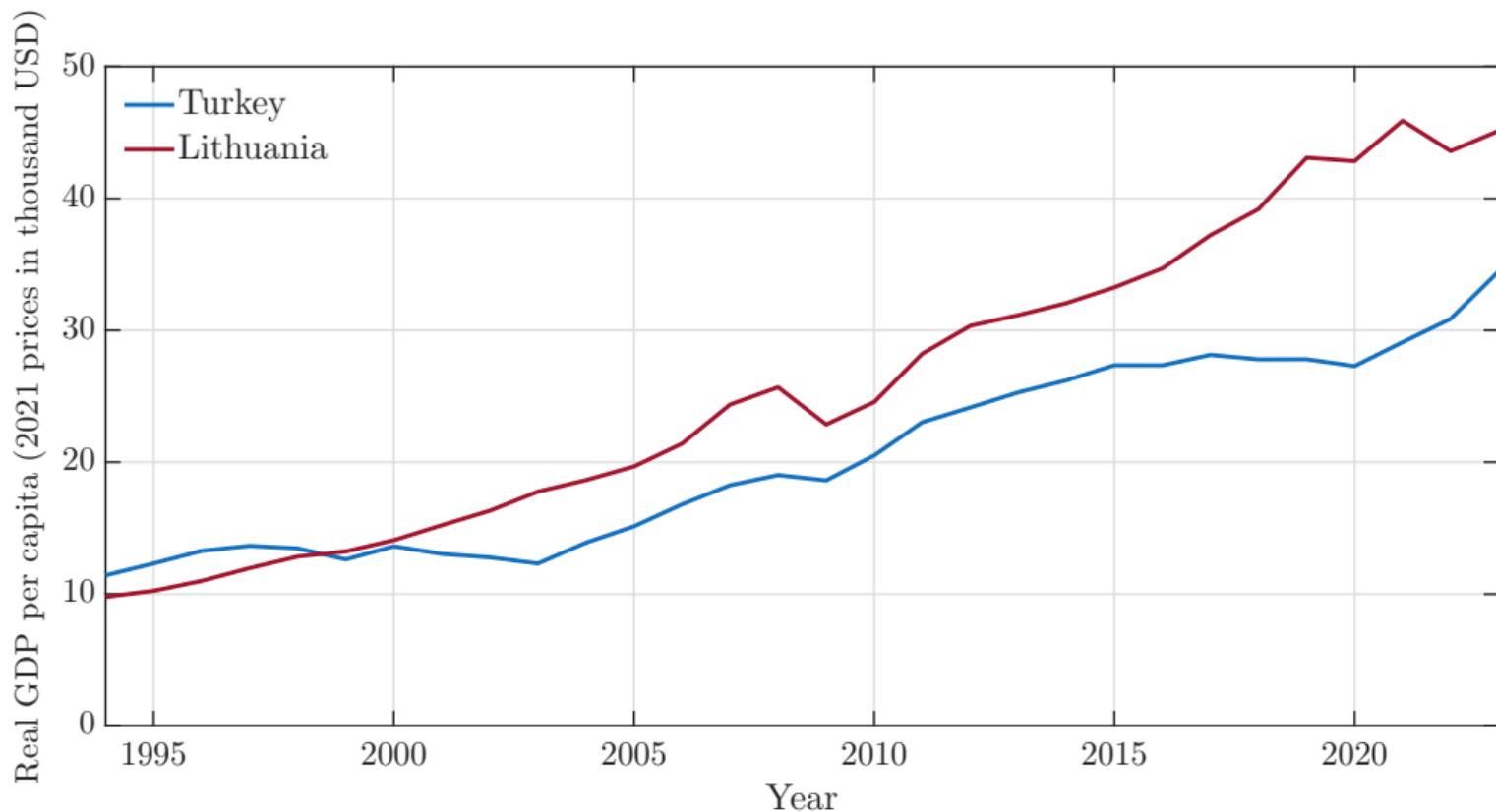
Real GDP: Turkey vs. Korea



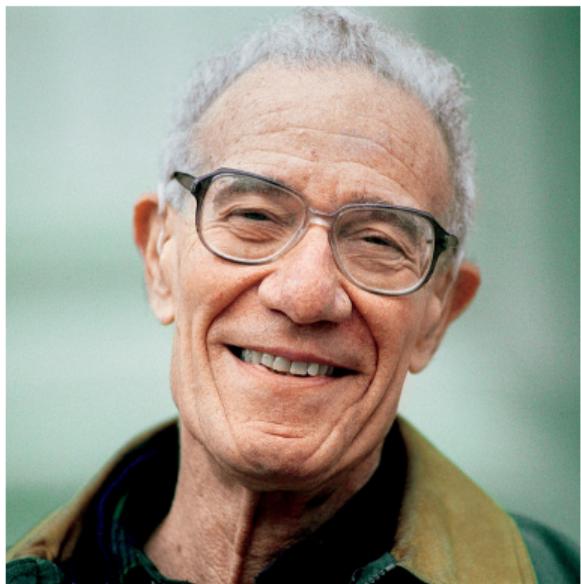
Real GDP per capita: Turkey vs. Korea



Real GDP per capita: Turkey vs. Lithuania



“Why are some countries richer than others?”



Robert M. Solow (1924-2023)

“A Contribution to the Theory of Economic Growth”

The Quarterly Journal of Economics,
Vol. 70, No. 1 (Feb. 1956), pp. 65-94

For the interested:

- “Robert Solow: An Affectionate Portrait” by Paul A. Samuelson
- Interview with Robert Solow by Douglas Clement (Fed Minneapolis)

Robert M. Solow:

“All theory depends on assumptions which are not quite true. That is what makes it theory. The **art** of successful theorizing is to make the inevitable **simplifying** assumptions in such a way that the final results are not very sensitive. *A ‘crucial’ assumption is one on which the conclusions do depend sensitively, and it is important that crucial assumptions be reasonably realistic.* When the results of a theory seem to flow specifically from a special crucial assumption, then if the assumption is dubious, the results are suspect.”



The Solow Model: In Words

- ▶ Output is produced using capital and labor (factors of production),
- ▶ A constant fraction of output is saved and invested in capital.
- ▶ Capital depreciates over time.
- ▶ Labor force size changes.
- ▶ Productivity improves over time.
- ▶ These forces determine how output per worker evolves.



The Solow Growth Model

$$\text{Production: } Y_t = F(A_t, K_t, L_t)$$

$$\text{Savings: } sY_t = I_t$$

$$\text{Investment and Depreciation: } K_{t+1} = K_t + I_t - \delta K_t$$

$$\text{Labor force change: } L_{t+1} = L_t + nL_t$$

$$\text{Productivity improvement: } A_{t+1} = A_t + aA_t$$

$$\text{Output per worker* evolution: } y_{t+1} = G(y_t, s, \delta, n, a)$$

⇒ Give me the initial conditions (A_0, K_0, L_0) , and the parameters (s, δ, n, a) ; I will tell you what happens over time.

The Solow Growth Model

$$\text{Production: } Y_t = F(A_t, K_t, L_t)$$

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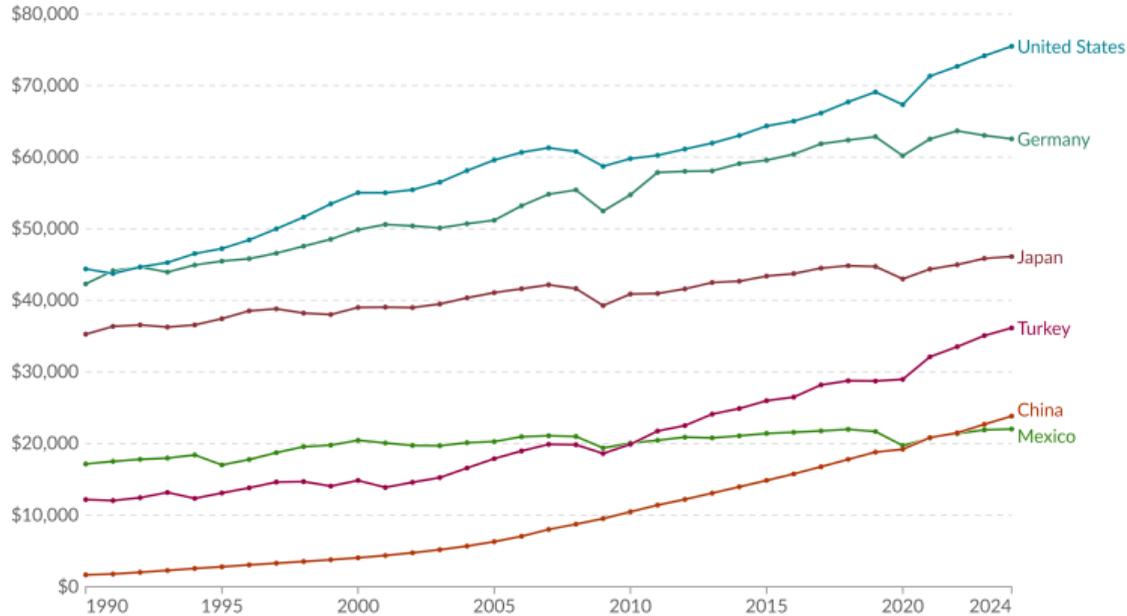
*Another simplification

Output per person

Our World
in Data

GDP per capita

GDP per capita is a country's gross domestic product¹ divided by its population. This data is adjusted for inflation and differences in living costs between countries.



Data source: Eurostat, OECD, IMF, and World Bank (2026)

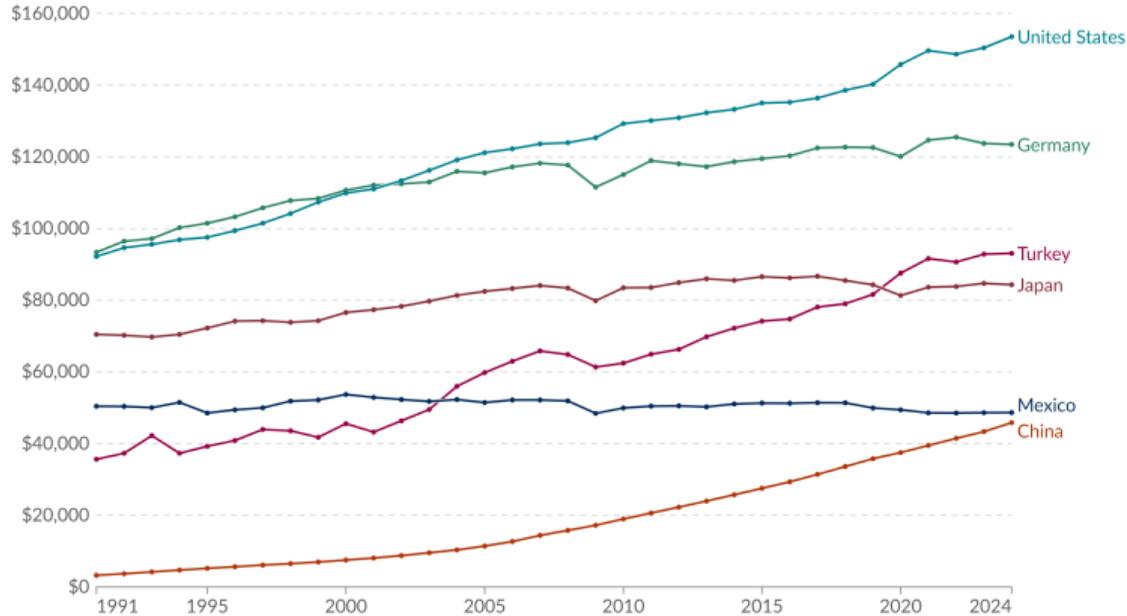
OurWorldinData.org/economic-growth | CC BY

Note: This data is expressed in international-\$³ at 2021 prices.

GDP per employed person, 1991 to 2024

Our World
in Data

Gross domestic product (GDP) divided by the total number of people employed in the economy. This data is adjusted for inflation and differences in living costs between countries.



Data source: World Bank staff estimates, via World Bank (2026)

OurWorldinData.org/economic-growth | CC BY

Note: This data is expressed in international-\$¹ at 2021 prices.

The Solow Growth Model

- ▶ Output is produced using capital and labor (factors of production), enhanced by productivity. Production: $Y_t = F(A_t, K_t, L_t)$
- ▶ A constant fraction of output is saved and invested in capital. Savings: $sY_t = I_t$
- ▶ Capital depreciates over time. Investment and Depreciation: $K_{t+1} = K_t + I_t - \delta K_t$
- ▶ Labor force size changes. Labor force change: $L_{t+1} = L_t + nL_t$
- ▶ Productivity improves over time. Productivity improvement: $A_{t+1} = A_t + aA_t$
- ▶ These forces determine how output per worker evolves.
Output **per worker** evolution: $y_{t+1} = G(y_t, s, \delta, n, a)$ where $y_t = Y_t/L_t$

The Solow Growth Model

Production: $Y_t = F(A_t, K_t, L_t)$

Savings: $sY_t = I_t$

Investment and Depreciation: $K_{t+1} = K_t + I_t - \delta K_t$

Labor force change: $L_{t+1} = L_t + nL_t$

Productivity improvement: $A_{t+1} = A_t + aA_t$

Output per worker evolution: $y_{t+1} = G(y_t, s, \delta, n, a)$

where $y_t = Y_t / L_t$

A simulation of the basic Solow model



“Why are some countries richer than others?”

- ▶ Solow model: “Productivity”
 - ▶ not a head start
 - ▶ not donations from large countries
 - ▶ not a small population

steady state (noun)

a condition of a process that does not change in time.

Source: Merriam–Webster Dictionary

- ▶ It is about the net change of some variable:

$$y^* = \frac{A_0 F(K_t, L_t) \uparrow}{L_t \uparrow}$$

- ▶ Abstract term. Higher \neq better for some variables:

$$\tilde{y}^* = \frac{F(K_t, A_t L_t)}{A_t L_t}$$

Production function

$$Y_t = F(A_t, K_t, L_t, H_t)$$

- ▶ Y_t : production,
- ▶ K_t : capital,
- ▶ L_t : labor,
- ▶ A_t : productivity,
- ▶ H_t : human capital

Alternative forms:

$$Y_t = A_t K_t^\alpha L_t^{1-\alpha} \quad 0 < \alpha < 1$$

$$Y_t = K_t^\alpha (A_t L_t)^{1-\alpha}$$

$$Y_t = A_t K_t^\alpha$$

$$Y_t = A_t K_t^\alpha (H_t L_t)^{(1-\alpha)}$$

The Solow Growth Model (No Productivity Growth)

$$\text{Production: } Y_t = K_t^\alpha L_t^{1-\alpha}$$

$$\text{Savings: } sY_t = I_t$$

$$\text{Capital accumulation: } K_{t+1} = (1 - \delta)K_t + I_t$$

$$\text{Labor force growth: } L_{t+1} = (1 + n)L_t$$

$$\text{Output per worker evolution: } y_{t+1} = G(y_t, s, \delta, n)$$

Steady state level of output per worker:

$$y^* = \left(\frac{s}{\delta + n} \right)^{\frac{\alpha}{1-\alpha}}$$

The Solow Growth Model (With Productivity Growth)

$$\text{Production: } Y_t = K_t^\alpha (A_t L_t)^{1-\alpha}$$

$$\text{Savings: } sY_t = I_t$$

$$\text{Capital accumulation: } K_{t+1} = (1 - \delta)K_t + I_t$$

$$\text{Labor force growth: } L_{t+1} = (1 + n)L_t$$

$$\text{Productivity growth: } A_{t+1} = (1 + a)A_t$$

$$\text{Output per worker evolution: } y_{t+1} = G(y_t, s, \delta, n, a)$$

Steady state level of output per effective worker:

$$\tilde{y}^* = \left(\frac{s}{\delta + n + a} \right)^{\frac{\alpha}{1-\alpha}} \quad \text{where } \tilde{y}_t \equiv \frac{Y_t}{A_t L_t}$$